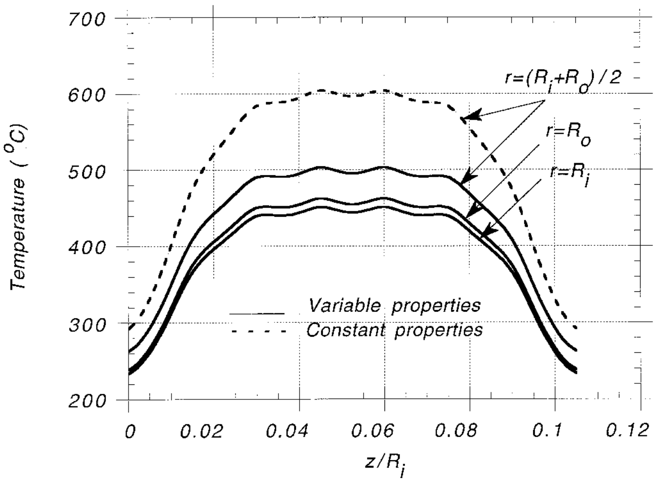

Fig. 2 Axial temperature profiles at fixed radial distances and time of 1 s.

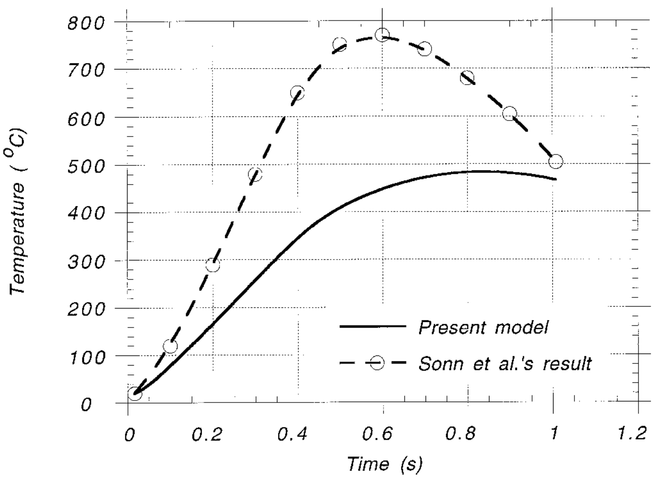

Fig. 3 Maximum temperature rise in the carbon–carbon composite disk brake assembly.

Discussion

Figure 2 shows the axial temperature profiles at three different radial distances and a fixed time of 1 s. Because of the cooling effect by convection, the inner and outer surfaces exhibit lower temperatures, while the maximum temperature profile lies somewhere between the midradius and the outer radius, because the frictional heat generation that is proportional to the radial distance increases toward the outer radius. The axial temperature distribution at midradius for the case of constant thermophysical properties is also included in Fig. 2 for comparison. When the thermophysical properties are evaluated at an average temperature of 300°C, a considerably high brake temperature variation is obtained. The difference in temperatures at 1 s can be as high as 20–30%. This indicates that variation of thermophysical properties with temperature is important and should be considered.

Figure 3 shows the temperature rise at the location where the temperature is maximum. A sharp increase in temperature is observed during the early time of braking action. The increase in temperature then slows down, and even a decrease in temperature can be obtained. This is the consequence of decreasing the angular speeds of the disks relative to one another and of the frictional heat generation associated with it. The history of the maximum temperature obtained using constant thermophysical properties as given by Sonn et al.[3] is also included in Fig. 3 for comparison.

The effect of h on the brake temperature variation has also been studied. It is found that the effect of h on the brake temperature is noticeable only in the vicinity of the outer surfaces where the brakes are exposed to convection. The internal areas of the disk brake assembly are not affected significantly with a change of h.

Conclusions

The transient temperature analysis of a multiple carbon–carbon aircraft disc brake system is obtained numerically using temperature-dependent thermophysical properties. It is shown that the specific heat is the main factor affecting the temperature distribution. An assumption of constant thermophysical properties can lead to 20–30% of inaccurate results. The maximum temperatures are obtained around the middle disk and at locations closer to the outer radius. The temperature rise is initially sharp and declines afterward. This is a result of time variations of angular velocity and pressure functions assumed in the analysis. The effect of convective heat transfer coefficient on the brake temperature variation is negligible.

Acknowledgment

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References

[1]Delmonte, J., *Technology of Carbon and Graphite Fiber Composites*, Van Nostrand Reinhold Co., New York, 1981, pp. 407–616.

[2]Currey, N. S., *Aircraft Landing Gear Design: Principles and Practices*, AIAA Education Series, AIAA, Washington, DC, 1988, pp. 148, 149.

[3]Sonn, H., Kim, C., Hong, C., and Yoon, B., "Axisymmetric Analysis of Transient Thermoelastic Behaviors in Composite Brake Disks," *Journal of Thermophysics and Heat Transfer*, Vol. 10, No. 1, 1996, pp. 69–75.

[4]Dowding, K., Beck, J., Ulbrich, A., Blackwell, B., and Hayes, J., "Estimation of Thermal Properties and Surface Heat Flux in Carbon–Carbon Composite," *Journal of Thermophysics and Heat Transfer*, Vol. 9, No. 2, 1995, pp. 345–351.

Exact Determination of Transient Cooling in Small Bodies by Nonlinear Natural Convection

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Nomenclature

A	=	surface area
Bi	=	Biot number, $\bar{h}L_c/k_s$
c	=	specific heat capacity
g	=	gravitational acceleration
$\bar{h}$	=	space–mean convection coefficient
K_{sf}	=	solid–fluid thermal conductivity ratio, k_s/k_f
k	=	thermal conductivity

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L_c = characteristic length
$\overline{Nu}_{L_c}$ = space–mean Nusselt number, $\bar{h}L_c/k_f$
Pr = Prandtl number, ν/α
$Ra_{L_c,i}$ = modified Rayleigh number, $g\beta_f(T_i - T_\infty)L_c^3/\nu_f\alpha_f$
$Ra_{L_c,w}$ = Rayleigh number, $g\beta_f(T_w - T_\infty)L_c^3/\nu_f\alpha_f$
T = space–mean temperature
t = time
V = volume
α = thermal diffusivity
β = isobaric coefficient of thermal expansion
θ = dimensionless T, $(T - T_\infty)/(T_i - T_\infty)$
ν = kinematic viscosity
ρ = density
τ = dimensionless t, $\alpha_s t/L_c^2$

Subscripts
ex, lin = exact, linearized
f, s, w = fluid, solid, surface

Introduction

THE analysis of transient cooling of solid bodies by conduction and convection to a fluid is governed by k_s, $\bar{h}$, and L_c. In turn, the cooling process can be rephrased by a dimensionless parameter, $Bi = \bar{h}L_c/k_s$. Conceptually, Bi embraces a ratio of an internal thermal resistance by conduction inside a solid body to an external thermal resistance by convection between the surface of the body and the surrounding fluid.[1]

Adopting a distributed-based model, the exact transient temperature distribution within regular bodies, e.g., slab, cylinder, and sphere, can be determined analytically by evaluating various infinite series for a range of Bi numbers: $0 < Bi < \infty$ (in theory). Conversely, for nonregular bodies, the transient temperature distribution has to be obtained numerically, preferably by the finite element method or the boundary element method.

In general, natural convective flows that are characterized by $\bar{h}$ are affected by T_w. Therefore, because of nonlinearities, several iterations seem to be necessary to predict the temporal responses, irrespective of the formulation and the computational procedure employed. In contrast, the equivalent approach involving forced convective flows is direct, because $\bar{h}$ is invariant with T_w when small temperature differences between the body and the fluid prevail.

One limiting condition for transient heat conduction in bodies cooled by convection corresponds to situations in which Bi is small, i.e., <0.1. An important application of this condition is associated with the generation of temperature–time curves for quenching processes in metallurgical applications.[2] Conceptually, this subset of unsteady heat conduction problems is indicative of a prevailing conductive thermal resistance that is minuscule compared to the convective thermal resistance. Suffice it to say that the criterion $Bi < 0.1$ is often satisfied by bodies that are small in size, and/or whose materials have large thermal conductivity, and/or which are exposed to weak convective environments. Logically, these inherent physical restrictions gave rise to an approximate but powerful model referred to as the lumped-based model.[1] This model relies on the idealization that transient conduction 1) takes place in a spatially, quasi-isothermal body and 2) is controlled by $\bar{h}$, which is unaffected by temperature.

The objective of this Note is to report an exact analytic solution of the nonlinear ordinary differential equation (ODE) that regulates the natural convective cooling of regular and irregular bodies governed by $Bi < 0.1$. Essentially, the nonlinearity arises from the variation of $\bar{h}$ with T in the Rayleigh number.

Mathematical Analysis

Natural Convective Cooling of Simple Bodies

Consider the transient cooling of a simple solid body of volume V and surface area A possessing a uniform initial temperature T_i, at $t < 0$. At $t = 0$, the surface of the body enters in contact with a natural convective flow of a fluid whose T_∞ is also uniform, but different from T_i. The influence of thermal radiation between the surface of the body and the fluid is neglected. Oils are typical examples of fluids that satisfy this condition.[2] The thermophysical properties of the body material and the fluid are independent of temperature.

The lumped-based model leads to a first-order ODE:

$$\frac{dT}{dt} = -\frac{\bar{h}A}{\rho_s c_s V}(T - T_\infty), \qquad T(0) = T_i \tag{1}$$

The customary procedure for integrating Eq. (1) presupposes that $\bar{h}$ is temperature-independent for any type of convective cooling (either forced or natural). Unquestionably, this approximate route is advantageous because it produces a simple ODE that is not only linear but, more important, is separable.

When convective cooling occurs by natural currents, $\bar{h}$ becomes a function of T; consequently, the computed temperature results based on a constant $\bar{h}$ may lead to erroneous temperature–time curves in quenching operations.[2] In view of the foregoing, a different strategy will be pursued and implemented in the following paragraphs, retaining the realistic temperature dependency of $\bar{h}$ in Eq. (1). The introduction of dimensionless variables for temperature and time, together with the space–mean Nusselt number and the solid–fluid thermal conductivity ratio, enables the transformation of Eq. (1) into

$$\frac{d\theta}{d\tau} = -\frac{1}{K_{sf}}[\overline{Nu}_{L_c}(\theta)]\cdot\theta, \qquad \theta(0) = 1 \tag{2}$$

From scale analysis,[3] natural convective heat transfer for isothermal bodies has been traditionally represented by $\overline{Nu}_{L_c}$, which, in turn, is affected by $Ra_{L_c,w}$ (w implies dependency on T_w) and Pr. Correspondingly, experimental data for simple bodies have been conveniently expressed by a generalized correlation equation[4]:

$$\overline{Nu}_{L_c} = C_1 + \frac{C_2 Ra_{L_c,w}^{1/4}}{g(Pr)} \tag{3}$$

where the constants C_1 and C_2 are inherent to the shape of the bodies. Also, $g(Pr)$ is slightly susceptible to the shape of the body.

Fundamentally, during the transient cooling of a body, T_w in Eq. (3) changes continuously with time, thereby $Ra_{L_c,w}$ also varies continuously with time. To avoid the latter variation, it is beneficial to introduce a new $Ra_{L_c,i}$ that relies on T_i, a fixed quantity. Because this temperature is invariant with time, $Ra_{L_c,i}$ becomes a unique number able to describe the entire transient cooling process. Therefore, the relation between both Rayleigh numbers is expressed as

$$Ra_{L_c,w} = Ra_{L_c,i}\theta \tag{4}$$

Hence, Eq. (2) can be converted into

$$\frac{d\theta}{d\tau} = -\frac{C_3}{K_{sf}}\theta - C_4\frac{Ra_{L_c,i}^{1/4}}{K_{sf}g(Pr)}\theta^{5/4}, \qquad \theta(0) = 1 \tag{5}$$

where C_3 and C_4 are new constants.

Natural Convective Cooling of Nonsimple Bodies

Yovanovich[5] discovered that natural convection heat transfer from bodies of other regular shapes follows a well-established Nusselt–Rayleigh relationship similar to that of simple bodies.[4] The need for simpler formulas was his motivation to develop a universal correlation equation containing implicitly the limit for pure conduction, $Ra_w \to 0$. Yovanovich[5] proposed the square root of the body surface as the length scale, and this aspect constituted a key element in the derivation. The correlation equation

$$\overline{Nu}_{\mathscr{L}} = \overline{Nu}^0_{\mathscr{L}} + \frac{0.67 G_{\mathscr{L}} Ra^{1/4}_{\mathscr{L}}}{g(Pr)} \tag{6}$$

where $\mathscr{L} = \sqrt{A}$, involves two constants and applies to short cylinders, spheres, various orientations of cubes, spheroids, etc. The numerical values of the constants are listed in Ref. 5. Note that the structure of Eqs. (3) and (6) is identical.

Analytical Integration

A special type of first-order ODE is Bernoulli's equation[6]

$$\frac{d\theta}{d\tau} = -a\theta - b\theta^p, \qquad \theta(0) = 1 \tag{7}$$

where p is a constant, nonnegative parameter different from 1. The presence of θ^p in Eq. (7) prevents it from being linear.

Equation (5) can be embodied into Eq. (7), and its analytic solution can be taken directly from[6]

$$\theta(\tau) = \frac{a^4}{[(a + b)\exp(a\tau/4) - b]^4} \tag{8}$$

In view of the previous text, it has been verified that as long as $Bi < 0.1$ and surface radiation is minimal (opaque fluids), accurate temporal variations of the space–mean temperature, θ, of regular (simple and nonsimple) as well as irregular bodies can be determined analytically at any time level. Numerical evaluation of Eq. (8) is easy, and its level of difficulty is comparable to the one pertinent to uniform space–mean convection coefficients occurring in forced convection cooling.

For quenching applications, the analytical nature of Eq. (8) is advantageous for purposes of differentiating the temperature–time function. It is widely known that cooling curves, θ vs τ, are useful to metallurgical engineers because the cooling rate in a test specimen can be estimated throughout the entire quenching cycle.[2] Moreover, from the cooling curve, it is possible to calculate other relevant variations of the cooling rate curve, such as changes of the cooling rate as a function of either time or temperature.[2]

Practical Example: Quenching of a Sphere

An iron sphere 2 cm in diameter is removed from a furnace at 220°C and quenched in an oil bath at 20°C. Calculate the relative errors in the cooling curves when the temperature distributions are computed with 1) variable $\bar{h}$ and 2) constant $\bar{h}$.

As a first step, we obtain the value of $Bi = 0.01$. Because $Bi < 0.1$, the temperature is uniform throughout the sphere and the lumped-based model is applicable. The correlation Eq. (3), wherein $C_1 = 2$ and $C_2 = 0.589$, can be employed for case 1. The linearization necessary in case 2 requires the replacement of the exponent $\frac{5}{4}$ with 1 in Eq. (5), so that the linearized temperature solution simplifies to the standard form $\theta = \exp[-(a + b)\tau]$.

The linearized cooling curve depending on constant $\bar{h}$ lies below the exact cooling curve based on variable $\bar{h}$. A comparison between them reveals no significant changes in the early stages of cooling, showing acceptable margins of error up to $\tau = 40$, e.g., $T_{ex}(40) = 86°C$ and $T_{lin}(40) = 82°C$, with an error of -5%. In the latter stages of cooling, the influence of variable $\bar{h}$ becomes notorious, and consequently the temperature errors caused by the linearization of $\bar{h}$ are more pronounced for large τ, i.e., $T_{ex}(80) = 48°C$ and $T_{lin}(80) = 38°C$, with an error of -21%.

References

[1]Luikov, A. V., *Analytical Heat Diffusion Theory*, Academic, New York, 1968.

[2]Totten, G. E., Bates, C. E., and Clinton, N. A., *Handbook of Quenchants and Quenching Technology*, ASM International, Materials Park, OH, 1993.

[3]Lienhard, J. H., *A Heat Transfer Textbook*, 2nd ed., Prentice–Hall, Englewood Cliffs, NJ, 1987.

[4]Churchill, S. W., "Free Convection Around Immersed Bodies," *Heat Exchanger Design Handbook*, edited by E. U. Schlünder, Hemisphere, New York, 1983, Sec. 2.5.7.

[5]Yovanovich, M. M., *On the Effect of Shape, Aspect Ratio, and Orientation upon Natural Convection from Isothermal Bodies of Complex Shape*, Vol. 82, Heat Transfer Div., American Society of Mechanical Engineers, New York, 1987, pp. 121–129.

[6]Polyanin, A. D., and Zaitsev, V. F., *Handbook of Exact Solutions for Differential Equations*, CRC Press, Boca Raton, FL, 1995.